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Irreducible Multiplier Corepresentations of the Extended Poincaré Group

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Abstract. The irreducible multiplier corepresentations of the extended Poincaré group $\mathscr{P}$ are, for positive and zero mass, determined by generalized inducing from a generalized little group. This approach is compared with the previous one of Wigner. For $m>0$, and any spin j, a particular realization is noted which is manifestly covariant on all four components of $\mathscr{P}$. The choice of covering group for $\mathscr{P}$ is discussed, and reasons are given for preferring a group for which S and T generate the quaternion group of order 8.

§ 1. Introduction

1.1. In this paper we consider, following Parthasarathy [7], Lever [4] and Shaw and Lever [10], a new approach* to the problem of determining all the physically relevant irreducible *multiplier corepresentations* (see, for example, [10]) of the extended Poincaré group $\mathscr{P}$ (and hence of determining the corresponding irreducible PUA-representations – see [7] – of $\mathscr{P}$). By "physically relevant" we mean those representations such that $p^2 \geqq 0, p_4>0$ and, in the case $p^2=0$ of zero mass, such that the spin is finite.

As all physicists know, the positive energy condition $p_4>0$ entails that time reversal T and space-time inversion $ST=-I$ must be represented by antiunitary operators, and space inversion S by a unitary operator. In other words, in the terminology of [7], we consider only those PUA-representations associated with the particular UA-decomposition

$$\mathscr{P}=\mathscr{P}^{\uparrow} \cup \mathscr{P}^{\downarrow} . \tag{1.1}$$

In this paper we will not at all discuss the problems (see [14], [3]) of the physical interpretation or existence of the discrete symmetry operators. Our object instead is to clarify the possible mathematical approaches to the problem alluded to in the opening paragraph. In particular we will describe a new method of attack on the problem which

* *Note Added in Proof:* See [15] for a simplified account of the approach in the case of non-zero mass.