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The Bloch Equations

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Abstract. We consider a spinor interacting with a heat bath of harmonic oscillators in equilibrium and we prove that the phenomenological Bloch equations for time development are satisfied exactly if the spin is $\frac{1}{2}$ and to first order in the inverse temperature if the spin exceeds $\frac{1}{2}$.

§ 1. Introduction

In 1946 Bloch [1] proposed the differential equation

$$\frac{d\boldsymbol{M}}{dt} = \gamma \boldsymbol{M} \times \boldsymbol{H} - \frac{M_1}{T_2}\boldsymbol{e}_1 - \frac{M_2}{T_2}\boldsymbol{e}_2 - \frac{(M_3 - M_0)}{T_1}\boldsymbol{e}_3 \tag{1}$$

for the time dependence of the macroscopic nuclear polarization $M(t)$ under the influence of an external magnetic field $\boldsymbol{H}$. $\gamma = \mu/jh$ is the gyromagnetic ratio of the nuclei under consideration with magnetic moment μ and spin j. The constants T_1 and T_2 are the longitudinal and transverse relaxation times respectively. Later Bloch and Wangness [2] attempted to justify these phenomenological equations theoretically with the simplifying assumption that the nucleus under consideration reacts independently of the other nuclei. In this paper we consider a fully quantum mechanical model by replacing the electromagnetic field by an infinite heat bath of harmonic oscillators in equilibrium keeping the simplifying assumption in [2]. We show that if $j = \frac{1}{2}$, our model, described in § 2, satisfies an equation of the same form as (1) in the weak coupling limit if the time is rescaled; we find also that if j exceeds $\frac{1}{2}$, to first order in the inverse temperature the model satisfies an equation of the form of (1). In § 3 we obtain an equation for the time development of an observable while in § 4 we obtain the Bloch equations by taking the spin as the observable.

This work is very similar to [4] which considers a harmonic oscillator interacting with a heat bath of harmonic oscillators though in the latter, expressions are simpler due to the fact that the particle is of the same type as those of which the bath is composed. We have used the description of the infinite heat bath provided in [4].