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Relativistic Quantum Statistical Mechanics in Two-Dimensional Space-Time

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Abstract. We construct for a boson field in two-dimensional space-time with polynomial or exponential interactions and without cut-offs, the positive temperature state or the Gibbs state at temperature $1/\beta$. We prove that at positive temperatures i.e. $\beta < \infty$, there *is no phase transitions and the thermodynamic limit exists and is unique for all interactions.* It turns out that the Schwinger functions for the Gibbs state at temperature $1/\beta$ is after interchange of space and time equal to the Schwinger functions for the vacuum or temperature zero state for the field in a periodic box of length β, and the lowest eigenvalue for the energy of the field in a periodic box is simply related to the pressure in the Gibbs state at temperature $1/\beta$.

1. Introduction

Although the study of the statistical mechanics for quantum systems has made good progress the last ten years [1], the progress has been best for discrete systems of lattice systems. The main difficulty in connection with continuous systems has been that the group of time automorphisms α_t for the Schrödinger particles is non local. The consequence of this non locallity is that the infinite system of interacting Schrödinger particles do not agree well with the generally accepted picture of a quantum statistical mechanics described in terms of a local C^*-algebra or a C^*-algebra of local operators, on which the time acts as a group α_t of C^*-automorphisms. Hence we get a somewhat discouraging situation, that the only known realistic model of a statistical quantum mechanics, namely the system of interacting Schrödinger particles, does not conform to the highly developed abstract theory of quantum statistical mechanics.

For this very reason the question of studying relativistic particles instead of Schrödinger particles appears quite natural, since in any relativistic theory there should be an upper bound for the propagation speed and this would force the group of time automorphisms α_t to be local. This is the motivation for this paper.

We know free relativisic particles are described by free quantum fields and it is a general belief that interacting relativistic particles are