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# Stability and Equilibrium States

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**Abstract.** For an infinite dynamical quantum system idealized as a $C^*$-algebra acted upon by time-translations automorphisms in an asymptotically abelian way, we propose to characterize equilibrium states by the three properties of stationarity, stability for local perturbations of the dynamics, and relative purity. We show that a state with these properties either gives rise to a one-sided energy spectrum or is a KMS- (i.e. essentially a limit Gibbs-) state.

## I. Introduction

The problem of characterizing equilibrium states in statistical mechanics is traditionally treated in the following steps:

1) One argues that the equilibrium value of an observable (in an isolated system of finite size) corresponds to the average value in a microcanonical ensemble of such systems.

2) One considers the thermodynamic limit, increasing the size of the system to infinity. It is in this limit that the fluctuations within the microcanonical ensemble become zero for intensive quantities and the thermodynamic laws become true for the individual system. Moreover, in this limit various ensembles (such as the microcanonical, the canonical, the uniform ensemble) become equivalent in the sense that they all give the same expectation value for local observables.

Let us call the expectation functional over the local observables which is obtained in the thermodynamic limit from any of the above mentioned ensembles a "limit Gibbs' state". The limit Gibbs states of a quantum system have an interesting property, first pointed out by Kubo [1] and by Martin and Schwinger [2] which may serve to compute these states without considering finite systems and limiting procedures. The Kubo-Martin-Schwinger (KMS)-condition, formulated algebraically in [3], has been the subject of much study in recent years by physicists and mathematicians.

From the point of view of physics, this work has strengthened the belief that the KMS-condition gives indeed an adequate characterization of equilibrium states in an infinite quantum system. On