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New Types of Singularity in General Relativity: The General Cylindrically Symmetric Stationary Dust Solution

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Abstract. The general solution of Einstein's equations for a stationary cylindrically symmetric distribution of pressure-free matter is obtained. It contains a function which may be freely prescribed. Using this freedom examples are given of new types of singularity in General Relativity.

1. Introduction

Recent work [1, 2] on the problem of singularities in relativistic cosmological models has made it clear that a variety of types of singularity may occur, such as Ricci curvature ("big bang") singularities and intermediate singularities ("whimpers"). In [1] further types were discussed, including *oscillating Ricci singularities* and *Weyl (conformal) singularities*, but no exact examples could be given. We aim here to provide such examples, so we begin by making precise the notions of "oscillating Ricci" and "Weyl" singularities.

In an inextensible spacetime the existence of a singularity is deduced from that of an inextensible geodesic (or more generally an inextensible curve) having finite length as measured by a generalised affine parameter. Let the curve be $\chi(v)$, defined for $0 \leqq v \leqq v_+$, and suppose it cannot be extended beyond v_+. Consider the components $R_{abcd}(v)$ of the Riemann tensor in orthonormal bases $\{e_a\}$ defined along $\chi(v)$. Both oscillating Ricci singularities and Weyl singularities are types of *curvature singularities*: these occur when there is *no* orthonormal basis defined along $\chi(v)$ such that all $R_{abcd}(v)$ tend to finite limits as $v \to v_+$. The division into Ricci and Weyl singularities comes from considering which part of the Riemann tensor is failing to tend to a limit. If the Ricci tensor components do not tend to finite limits in any orthonormal frame we have a *Ricci singularity*, and this is the familiar "big bang" singularity if some components actually diverge in every such frame. If a Ricci singularity is such that in some orthonormal frames some components $R_{ab}(v)$ oscillate finitely (but do not tend to a limit) and the rest do tend