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Was the Big Bang a Whimper?

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Abstract. In many cases the spatially homogeneous cosmological models of General Relativity begin or end at a "big bang" where the density and temperature of the matter in the universe diverge. However in certain cases the spatially homogeneous development of these universes terminates at a singularity where all physical quantities are well—behaved (a "whimper") and an associated Cauchy horizon. We examine the existence and nature of these singularities, and the possible fate of matter which crosses the Cauchy horizon in such a universe. The nature of both kinds of singularity is illustrated by simple models based on two-dimensional Minkowski space-time; and the possibility of other types of singularity occuring is considered.

1. Introduction

In the standard spatially homogeneous and isotropic cosmological models of General Relativity (see e.g. Refs. [1, 2]) a singularity necessarily develops at the beginning of an expansion phase if the energy density μ and pressure p of the matter obey the inequality

$$\mu + 3p \geqq 0 \tag{1.1}$$

at all times. (We shall here assume the cosmological constant Λ is zero; if it is non-zero, the corresponding inequality is $\mu + 3p \geqq 2\Lambda$, where notation is as in [2].) Further this singularity is necessarily a physical one if either of the sets of inequalities

$$p \geqq 0\,, \quad \mu_0 > 0 \tag{1.2a}$$

or

$$1 \geqq dp/d\mu \geqq 0\,, \quad \mu_0 + p_0 > 0\,, \quad \mu_0 + 3p_0 > 0 \tag{1.2b}$$

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