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Uniqueness of the Positive Energy Parafermion Field

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Introduction

In 1953, Green [1] proposed that the usual anticommutation relations of Fermi-Dirac statistics be generalized. He proposed new relations which are satisfied by the Fermi-Dirac creation and annihilation operators but admit additional representations, each of which can also be given a particle interpretation. These particles are called parafermions.

For n degrees of freedom it has been shown [4] that the representations of Green's relations are just the representations of $SO(2n+1, R)$, which are well known. For infinitely many degrees of freedom the situation is more complicated. Reducible representations may be constructed by the use of Green's ansatz [1; 2] from which certain standard irreducible representations are chosen. The uniqueness of these standard representations is usually discussed in terms of the existence of a unique vacuum vector which is annihilated by all annihilation operators. The purpose of this paper is to obtain uniqueness results relating to unitary invariance as was done for ordinary particles by Segal [5, Theorems 1, 2, 3] and Weinless [6, Theorem 2.1].

Preliminaries

A bounded representation of Green's parafermion relations is a triple $\{H, C, K\}$ where H and K are complex Hilbert spaces and C is a complex linear map from H into the bounded operators on K satisfying

$$[[C^*(z), C(y)], C(x)] = -2 \langle x, z\rangle C(y)\,, \tag{1}$$

$$[[C(z), C(y)], C(x)] = 0 \qquad x, y, z \in H\,. \tag{2}$$

$C^*(z)$ represents the adjoint of $C(z)$.