Commun. math. Phys. 38, 47—63 (1974)

Analytic Properties of the Teukolsky Equation*

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Received January 2, 1974

Abstract. The analytic properties of the solutions to the Teukolsky equation in the complex frequency plane are investigated. The scattering coefficient Z_{in} is found to be an analytic function of the frequency except at singularities and at certain branch points in both the upper and lower frequency plane. The implications for the proof of the stability of the Kerr geometry given by Press and Teukolsky are discussed.

I. Introduction

In 1972 Teukolsky [1] derived a separable wave equation whose solutions describe the dynamical gravitational, electromagnetic, scalar, and neutrino field perturbations of a Kerr rotating black hole. Press and Teukolsky [2] have applied this equation to an investigation of the stability of the Kerr geometry under small gravitational perturbations. Their analysis depends on an understanding of the analytic properties of the solutions to the Teukolsky equation in the complex frequency plane. In this paper we will derive the relevant analytic properties.

The Teukolsky equation is a linear equation for a perturbation quantity ψ of spin-weight s. In the case of gravitational perturbations, for example, ψ would be the component ψ_0 or ψ_4 of the Weyl tensor in Newman-Penrose notation. In Boyer-Lindquist coordinates, the equation can be separated by breaking ψ into components which vary harmonically in time $[\exp(-i\omega t)]$ and by expanding it in terms of spheroidal harmonics ${}_s\mathcal{S}_l^m(\theta, \varphi)$ of spin-weight s. The resulting equation for the radial part R of ψ describing the perturbations of a Kerr geometry of mass M and specific angular momentum a takes the form

$$\frac{1}{\Delta^s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR}{dr}\right)+\left[\frac{K^2-2is(r-M)K}{\Delta}+4is\omega r-\lambda\right]R=0 \quad (1.1)$$

where $K=(r^2+a^2)\omega-am$, $\Delta=r^2-2Mr+a^2=(r-r_+)(r-r_-)$, and $\lambda=a^2\omega^2-2am\omega+A(a\omega)$. Here, $A(a\omega)$ is the separation constant of the

* Supported in part by the National Science Foundation.