Commun. math. Phys. 37, 183—191 (1974)

An Existence Proof for the Hartree-Fock Time-dependent Problem with Bounded Two-Body Interaction

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Received September 19, 1973; in revised form January 4, 1974

Abstract. Using fixed point theorems for local contractions in Banach spaces, an existence and uniqueness proof for the Hartree-Fock time-dependent problem is given in the case of a finite Fermi system interacting via a bounded two-body potential. The existence proof for the "strong" solution of the evolution problem is obtained under suitable conditions on the initial state.

1. Introduction

In general, starting from a quasi-free (or generalized-free) state ϱ of a finite or infinite Fermi system at the time $t = t_0$, the natural evolution of the system gives rise to a state ϱ_t which does not remain quasi-free for $t > t_0$, and trustworthy methods of successive approximations for solving the evolution problem except in trivial cases are not known. An approximate procedure for solving this problem is provided by the time-dependent Hartree-Fock theory, first obtained by Dirac [1] and afterwards generalized by Bogoliubov [2] and Valatin [3]. These equations can be obtained by considering the evolution of the one-particle density matrix T and assuming that ϱ_t remains quasi-free in a given time interval. Perturbative solutions of such equations for superconducting systems have been studied by Di Castro and Young [4].

In spite of the simplicity of the approach, the equation of motion for the one-particle density matrix T is non-linear so that the existence problem is not easy even in the most simple physical cases. Written in