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Haag-Ruelle-Hepp Scattering Formalism for Essentially Local Non-Localizable Fields

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Abstract. We generalize the axiom of locality to the non-localizable case by an axiom of essential locality which means a natural local continuity property of the field commutator on the light-cone. For localizable fields essential locality coincides with ordinary locality while localizable local fields restricted to non-localizable test spaces still remain essentially local. Essential locality is proved to be sufficient for a derivation of the usual Haag-Ruelle-Hepp scattering formalism. Functions of the free field are examples of essentially local fields which may not be localizable.

1. Introduction

The principal aim of the present paper is to give a generalization of the Haag-Ruelle-Hepp scattering formalism to the non-localizable case, i.e. to the case of test spaces not containing functions with compact support.

In the localizable case *causality*, usually introduced by local commutativity (in short: *locality*), is well known to be crucial for the derivation of the asymptotic conditions. Since the standard formulation of locality [20] does not apply to the non-localizable case, we have to look for some other suitable additional assumption.

Thus Steinmann demands the existence of generalized retarded products as defined in [18] and imposes certain regularity properties [19]. Steinmann himself quoted this a purely technical assumption which cannot be given any direct physical interpretation. Moreover, despite extensive research in this field, there does not exist any rigorous proof for the general existence of those Green's functions even in the tempered case (apart from the unrealistic case of two space-time dimensions [17]). It should be interesting, however, to investigate the possibilities of circumventing this difficulty by modification of the defining postulates, extracted from *sharp* Green's functions for localizable fields, which Steinmann introduced for convenience (compare [18] for this point).

Nevertheless we think it is desirable to work with a more direct substitute of locality which is applicable to both the localizable and the