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Ergodic Properties of an Infinite System of Particles Moving Independently in a Periodic Field*

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Abstract. We investigate the ergodic properties of a general class of infinite systems of independent particles which undergo nontrivial "collisions" with an external field, e.g. fixed convex barriers (the Lorentz gas). We relate the ergodic properties of these systems to the ergodic properties for a single particle moving in a finite box (with periodic boundary conditions) with the same dynamics. We prove that when the one particle system is mixing or a K-system for a sequence of boxes approaching infinity so is the infinite particle system with an equilibrium measure obtained as a Poisson construction over the one particle phase space.

1. Introduction

The ergodic properties of some "interesting" finite Hamiltonian systems are known in considerable detail [1, 2]. Very little is known, however, about the ergodic properties of nontrivial systems with an infinite number of degrees of freedom. These are of great interest for statistical mechanics. Sinai [3, 4], De Pazzis [5] and more recently, Aizenman, Goldstein, and Lebowitz [6] have investigated the ergodic properties of an infinite ideal gas in an arbitrary number of dimensions and of an infinite system of hard rods moving in one dimension while Lanford and Lebowitz [7] have considered the infinite harmonic system. While these systems have very good ergodic properties (K or Bernoulli systems) the physical interpretation of this result is that "local disturbances stream off to infinity where they are no longer visible" [8]. Thus these results shed little light on the "local approach to equilibrium" for macroscopic systems.

We investigate here the ergodic properties of some infinite systems with non-trivial "collisions", i.e., the transformation which occurs during a collision possesses itself good mixing properties. Except for these col-

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