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Martin-Dynkin Boundaries of Random Fields

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Abstract. Analogous of exit spaces of Dynkin [4] for Markov processes are constructed for random fields introduced by Dobrushin [2].

Let X be a finite set and let T be countable. Let $\Omega = X^T$ and let $\mathscr{B}_V$ be the σ-algebra generated by $\{\omega \in \Omega;\ \omega(t) = x\}_{t \in V, x \in X}$ for $V \subset T$. The σ-algebra $\mathscr{B}_T$ is denoted simply by $\mathscr{B}$. Let be given a system of *conditional distributions* $q_{V,\omega}(A)$, which satisfy the following conditions, where V is a finite subset of T, $\omega \in \Omega$ and $A \in \mathscr{B}_V$.

α) $q_{V,\omega}(\cdot)$ is a probability measure on $\mathscr{B}_V$.

β) $q_{V,\omega}(A)$ is a $\mathscr{B}_{V^c}$-measurable function of ω for $A \in \mathscr{B}_V$.

γ) If $V_1 \subset V_2$, then for $A \in \mathscr{B}_{V_1}$, $B \in \mathscr{B}_{V_2 \setminus V_1}$ and $\omega \in \Omega$

$$q_{V_2,\omega}(A \cap B) = \int_B q_{V_1, c(V_2;\omega',\omega)}(A)\, q_{V_2,\omega}(d\omega'),$$

where $c(V_2; \omega', \omega)(t) = \omega'(t)$ for $t \in V_2$, and $= \omega(t)$ for $t \notin V_2$.

A probability measure P on $(\Omega, \mathscr{B})$ is called a *random field* with conditional distribution q, if for $A \in \mathscr{B}_V$

$$P(A \mid \mathscr{B}_{V^c}) = q_{V,\omega}(A) \qquad \text{a.e. } (P).$$

Dobrushin [2] shows that the totality $\mathscr{P}$ of random fields with q is a non-empty, compact and convex set, if

δ) $\lim\limits_{W \to T} \sup\limits_{\omega'} |q_{V,\omega'}(A) - q_{V, c(W;\omega',\omega)}(A)| = 0$

for $A \in \mathscr{B}_V$ and $\omega \in \Omega$, which we assume throughout this note.

Let $V_1 \subset V_2 \subset \cdots$ be an increasing sequence of finite subsets V_n of T with $\cup V_n = T$. Let Ω_∞ be the set of ω for which there exists $\lim\limits_{n \to \infty} q_{V_n,\omega}(A)$ for every cylindrical A.

Let $Q_\omega(A)$ be the limit. $Q_\omega(\cdot)$ is countably additive on $\mathscr{B}_V$ for every finite subset V. Therefore it is extended to a probability measure on $\mathscr{B}$, which we denote by the same Q_ω. It is easy to see $Q_\omega \in \mathscr{P}$.

Let $\mathscr{B}_\infty = \bigcap\limits_V \mathscr{B}_{V^c}$, where V runs over the set of all finite subsets of T.