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Three Remarks on Axisymmetric Stationary Horizons

P. Hájiček

Institute for Theoretical Physics, University of Berne, Berne, Switzerland

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Abstract. In the first remark the catalogue of axisymmetric stationary horizons is completed by a subset of uncharged translation-symmetric horizons, which was ignored in the previous paper [5]. The subset consists of two one-parameter families: extreme Kerr horizon ($a = m$) and a more symmetric family. In the second remark the surface area, net angular momentum and net charges of a black hole are computed. It turns out that the four invariant functions A, B, C, D used in [5] to classify the horizons describe, at least formally and up to a constant factor: A the profile of the black hole surface, B the surface density of angular momentum and $C\cos D$, $C\sin D$ the surface density of electric and magnetic charge. In the third remark a simplified model of a black hole surrounded by a charged matter shell is found to satisfy a sort of generalized "no-hair-conjecture". An example of a non-Kerr-Newman field around a horizon is provided; the magnetic field in it is hoped to have some astrophysical importance.

1. Introduction

The search for black holes in close binaries (see, e.g. [1]) has shifted the attention from pure vacuum black holes to systems consisting of a hole surrounded with matter disks, rings and shells [2, 3].

If there is only vacuum or electrovacuum outside a black hole, then the well-known "no-hair-theorems" [4] tell us what the equilibrium (= stationary) state of the horizon and the field outside it will be: for static electrovacuum it is the Reissner-Nordström family and for stationary vacuum a two-parameter family which, if it contains the Schwarzschild solution, must be that of Kerr. No proof is attempted for stationary electrovacuum, but one conjectures that the Kerr-Newman solution is all what is possible.

A ring of matter surrounding the hole will deform the horizon and influence the field in its neighbourhood. In [5] (the last two papers of [5] will be denoted by I and II, and, e.g. the equation (x) of II by II (x)) an attempt was undertaken to determine what the equilibrium structure of such horizons and neighbouring fields can be. The method used was based on solving the characteristic initial data constraints for Einstein-Maxwell equations together with that part of Killing equations and their integrability conditions which concerns the inner structure of the