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Strict Convexity of the Pressure: A Note on a Paper of R. B. Griffiths and D. Ruelle

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Abstract. Strict convexity of the pressure of a quantum lattice gas is demonstrated in [1] with the help of a trace condition. An interpretation of that condition is given, and, simultaneously, an extension of the result of [1]. In particular, it is shown that the pressure is a continuous function of the lattice gas density.

I. Introduction

It has been demonstrated by Griffiths and Ruelle [1] that the pressure $P(\Phi_i)$ and the time automorphisms $\tau_t(\Phi_i)$, $i=1, 2$, exist are called function of the interaction Φ. One assumption used for the case of a quantum lattice gas is the following:

$$\mathrm{Tr}_Y \Phi(X)=0 \quad \text{for all} \quad Y \subset X, \quad \text{all finite} \quad X \subset \boldsymbol{Z}^\nu . \tag{1}$$

Here, $\boldsymbol{Z}^\nu$ describes the (ν-dimensional) lattice, Tr_Y denotes the partial trace. We are concerned with the interpretation of this condition which is not given in [1].

Definition 1.1. Two interactions Φ_1 and Φ_2 for which the pressures $P(\Phi_i)$ and the time automorphisms $\tau_t(\Phi_i)$, $i=1, 2$, exist are called physically equivalent if $P(\Phi_1)=P(\Phi_2)$ and $\tau_t(\Phi_1)=\tau_t(\Phi_2)$. We then write $\Phi_1 \simeq \Phi_2$.

In view of Theorem 2.2 below, this definition seems to be a sensible one. It will turn out that in every class of equivalent interactions with vanishing trace, there is a unique interaction with vanishing partial traces, i.e. satisfying (1), provided a certain temperedness condition is fulfilled. This allows a generalization of the results of [1]; in particular, we can show the continuity of the pressure as a function of the lattice gas density.

II. Notations and Results

We study a quantum lattice system over $\boldsymbol{Z}^\nu$, with a two-dimensional Hilbert space $\mathscr{H}_x$ attached to every $x \in \boldsymbol{Z}^\nu$, $\mathscr{H}_X=\bigotimes_{x\in X} \mathscr{H}_x$. $X, Y, \Lambda, \ldots$

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