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Ising Model in the High Density Limit

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Abstract. It is proved that the free energy per spin in the thermodynamic limit of an Ising model on a lattice with coordination number z approaches the classical Curie-Weiss free energy in the limit $z \to \infty$. The infinite spacial dimension limit of nearest neighbour lattice models is a special case of this result.

1. Introduction

It is well known that in a variety of circumstances one obtains the classical theories of phase transitions for fluid and magnetic systems in the limit of long range interactions. Such results to date (e.g. Lebowitz and Penrose [1], Thompson and Silver [2]) have been concerned with attractive pair potentials

$$\phi(\boldsymbol{r}) = \gamma^d v(\gamma|\boldsymbol{r}|) \tag{1.1}$$

of Kac type, in fixed spacial dimension d, in the limit $\gamma \to 0$.

Various expansions for thermodynamic quantities in powers of γ have been obtained [3, 4], and resummed in regions close to the critical point [5]. Prior to Refs. [1–5], Brout [6] obtained high density expansions for spin systems on lattices in inverse powers of the coordination number z of the lattice, with the classical Curie-Weiss theory as leading term. On the basis of Brout's work it has often been stated (e.g. [7]) that critical behaviour for spin systems on d-dimensional lattices ($z = 2d$ for a regular cubic lattice) should become classical in the limit $d \to \infty$.

Our purpose here is to prove that the $z \to \infty$ limit for an Ising model on a lattice with coordination number z indeed results in the classical Curie-Weiss theory. We stress that the lattice does not have to be regular or tied to a particular spacial dimension. All that is required is that each of the N points of the lattice be bonded or linked to z other points of the lattice.

The precise statement is as follows.

Theorem. *Let each point p of a lattice with N points and coordination number z be occupied by a spin $\mu_p = \pm 1$ and let the spins interact with*