Commun. math. Phys. 36, 91—114 (1974)

The Use of Reflection as Symmetry Operation in Connection with Peierls' Argument

Ole J. Heilmann

Department of Chemistry, H. C. Ørsted Institute, Copenhagen, Denmark

Received September 3, 1973; in revised form January 1, 1974

Abstract. It is shown that Peierls' argument can be extended to prove the coexistence of states which can be transformed into each other only by reflection, inversion or rotation. This makes Peierls' argument applicable to the repulsive lattice gas on a large class of lattices, as well as many models for liquid crystals, ferroelectrics etc. It is specifiquely shown how the argument can be applied to a lattice gas with nearest neighbour repulsion on the hexagonal lattice and the diamond lattice.

I. Introduction

The Peierls' argument is one of the few methods for proving the existence of phase transition without explicit calculation. The idea goes back to Peierls [1] and the method was given a rigorous formulation for the ordinary, ferro magnetic Ising model by Griffiths [2] and Dobrushin [3]. The argument has later been extended to other lattice models with similar properties [4] and more recently Ruelle [5] has succeeded in applying the method in the first rigorous proof of the existence of a phase transition in a continuous model.

The first application of Peierls' argument to lattice gases with repulsive potential is due to Dobrushin [6] who proved the existence of a phase transition for a lattice gas with nearest neighbour exclusion or nearest neighbour repulsion on a simple cubuc lattice in ν dimensions ($\nu \geqq 2$). The nature of the argument in the case of repulsion is somewhat different from the proof for systems with attractive forces. First, for the ferro magnetic Ising model one utilizes the symmetry between spin-up states and spin-down states in zero magnetic field at the crucial point of the proof; this makes the generalization from simple cubic lattices to any other lattice trivial. For the antiferromagnet one uses a spatial symmetry operation (see below) which is dependent on the structure of the lattice; therefore, this method is not generally applicabel and it should not be so since it is known that if the lattice is