Commun. math. Phys. 36, 59—72 (1974)


# On the Nature of "Strong" Bogoliubov Transformations for Fermions*

Gilles Labonté**
Theoretical Physics Institute, University of Alberta, Edmonton, Canada

Received November 6, 1973

**Abstract.** Properties of the unitary operator implementing a general Bogoliubov transformation on a system of fermions are examined. The nature of this operator (and that of the transformed vacuum state) is exhibited for the case in which the operators appearing in the "diagonal" part of the transformation do not have an inverse. This case turns out to be very different from either the case where the inverses exist or the case with Bogoliubov transformations on systems of bosons. In particular, such transformations can be unitarily implementable in the initial Fock-Hilbert space even though the new vacuum is orthogonal to the initial vacuum. Particles are then created with certainty (i.e. probability 1) in the initial vacuum state and even the new charge operator could be different from the initial one.

## Introduction

In many problems where a quantized field is acted upon by an external field, the differential equation of motion for the quantized field operator valued distribution is linear and homogeneous. This is the case, for example, when a Dirac electron-positron field, $\psi(t, x)$, is modified through minimal coupling with a classical electromagnetic field $A(t, x)$ according to

$$(-i\gamma \cdot \partial + m)\,\psi(t, x) = e\gamma \cdot A(t, x)\,\psi(t, x)\,.$$

In such problems, one is interested in knowing about the existence and nature of the $S$-matrix or of the unitary time evolution operator at finite times for the field operator variables or the one for the energy quanta creation and annihilation operator variables. Thus one is led to examine linear homogeneous relations, of the type described in Eq. (1.1) below[1],

* This work is part of the author's Ph. D. thesis done at the University of Alberta and was supported by the National Research Council of Canada and the Department of Advanced Education of the Government of Alberta.

** Research supported in part by Air Force Office of Scientific Research under Contract F 44620-71-C-0108 (USA).

[1] We will study here explicitly theories with distinct particles and antiparticles. The results for theories with only one kind of particle can be obtained in a similar fashion. They will be mentioned at the end.