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Some Aspects of Markov and Euclidean Field Theories

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Abstract. Various aspects of Markov field theory are treated. We give a Fock space description of the scalar free field with Nelson's Markov property formulated in terms of projections. We consider conditions imposed on analytically continued Wightman distributions at Euclidean points so that a Euclidean Markov field theory will result. Euclidean theories in higher dimensional imaginary times are considered. We show how the generalized free field theory can be interpreted as a Markov Euclidean field theory. The spatially cutoff linear perturbation model is solved in arbitrary space-time dimensions and the Wightman distributions are obtained explicitly in the limit as the cutoff is removed. The appendices contain a discussion and derivation of the Segal isomorphism and we give some generalizations of Feynman-Kac formulas in R^n and in the Fock space of Euclidean field theory.

Introduction

Euclidean field theory techniques, particularly Nelson's Markov property and consequent symmetry, have recently played an important role in constructive quantum field theory (see [1–4]). These methods are among the techniques used by Glimm and Spencer [5] to show that the Schwinger distributions for the spatially cutoff $P(\phi)_2$ model converge as the cutoff is removed, provided the coupling constant is sufficiently small. They obtain the Wightman distributions by analytic continuation and show that the corresponding relativistic quantum field theory has a mass gap. For earlier work on the relation of Euclidean and Minkowski field theories see Symanzik [6].

In this article we consider several aspects of Euclidean field theory. In Section I we describe the free scalar Euclidean field and formulate what we call the pre-Markov property of certain projections. This property is due to Nelson [1] and implies the Markov property – we isolate it because it does not require a probabilistic interpretation in its formulation, and because it can be used directly in many applications of the Markov property. Section II contains a brief discussion of the general