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# Fluctuating Magnetic Fields

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**Abstract.** Some problems pertaining to the behaviour of a classical spin under the influence of a random Gaussian magnetic field are discussed. It is shown that, in agreement with simple expectations, the magnetic moment is effectively decreased to lowest order. Various physical applications and connections with group theory are pointed out.

## 1. Introduction

The present investigation was motivated by the quest for a simple explanation of the size and sign of lowest order radiative corrections. It has often been proposed to understand qualitatively these effects by assuming that test charges be submitted in addition to external fields to random ones arising from vacuum quantum fluctuations [1]. Applied to a charged non-relativistic particle moving in a fixed external potential and to a fluctuating electric field, one is led to a formula for the Lamb shift in striking qualitative agreement with the exact result. A similar calculation performed for the anomalous magnetic moment yields a value of the correct order of magnitude but with *wrong* sign. This was generally attributed to added fluctuations arising from a relativistic treatment including negative energy states of the Dirac electron. Koba [2] pointed out that these added contributions were in the right direction. The fact that fluctuating magnetic fields tend to reduce the effective magnetic moment is suggested by the following heuristic argument. Consider the coupling of the magnetic moment $\boldsymbol{\mu}$ to the external field $\boldsymbol{B}_0$:

$$-\boldsymbol{\mu}\cdot\boldsymbol{B}_0 = -|\boldsymbol{\mu}|\,|\boldsymbol{B}_0|\cos\theta\,,$$

we can write $\cos\theta$ as $\cos\bar{\theta}\cos\tilde{\theta}+\sin\bar{\theta}\sin\tilde{\theta}\cos(\bar{\varphi}-\tilde{\varphi})$, where $\bar{\theta}$ and $\bar{\varphi}$ are the polar angles of the mean direction of $\boldsymbol{\mu}$ and $\tilde{\theta}$, $\tilde{\varphi}$ represent fluctuations. Averaging $\langle\cos\theta\rangle$ using the fact that $\tilde{\varphi}$ is uniformly distributed and that fluctuations are small, yields

$$\langle\cos\theta\rangle \cong \cos\bar{\theta}\left(1-\frac{\langle\tilde{\theta}\rangle}{2}\right).$$

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