

# The Equilibrium States of the Free Boson Gas

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**Abstract.** The generating functional of the cyclic representation of the canonical commutation relations for the equilibrium state of the free Boson gas is calculated, using a method due to Kac, as the thermodynamic limit of the grand canonical generating functional. The relation to the work of Araki and Woods is discussed.

## § 1. Introduction

We compute the generating functional of the cyclic representation of the canonical commutation relations corresponding to the equilibrium state of the free Boson gas. We use a method suggested by Kac [1] in his rigorous derivation of the London-Placzek formula for the pair distribution function, to obtain the thermodynamic limit of the grand canonical ensemble. This method enables us to treat a range of boundary conditions and to display their effect on the generating functional.

In § 2 we review the results about representations of the CCR which we shall require, and in § 3 we describe the procedure for passing to the thermodynamic limit and state as Theorem 1 our main result which describes the limiting form of the generating functional. A special case of our result has been announced by Weiringa [2]. As a step towards the proof of Theorem 1 we need to establish the existence of condensation; the essential result is stated as Theorem 2. In § 4 we sketch some applications of these results before proving them in § 5. We discuss the relation with the work of Araki and Woods [3] in § 6 where we construct the cyclic representation of the CCRs corresponding to the generating functional described in Theorem 1, and discuss some of its properties.

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