Commun. math. Phys. 35, 257—264 (1974)

On the Time Evolution Automorphisms of the CCR-Algebra for Quantum Mechanics

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Received September 13, 1973

Abstract. In ordinary quantum mechanics for finite systems, the time evolution induced by Hamiltonians of the form $H = \frac{P^2}{2m} + V(Q)$ is studied from the point of view of *-automorphisms of the CCR C^*-algebra $\bar{\Delta}$ (see Ref. [1, 2]). It is proved that those Hamiltonians do not induce *-automorphisms of this algebra in the cases: a) $V \in \bar{\Delta}$ and b) $V \in L^\infty(\mathbb{R}, dx) \cap L^1(\mathbb{R}, dx)$, except when the potential is trivial.

I. Introduction

Consider the Hilbert space $\mathscr{H} = L^2(\mathbb{R}^n, dx^n)$ of square integrable functions on $\mathbb{R}^n$. For notational convenience we restrict ourselves to the case $n = 1$. The general case is a trivial extension.

Define the Schrödinger position and momentum operators respectively by: for $\phi \in \mathscr{H}$, $x \in \mathbb{R}$.

$$(Q\phi)(x) = x\,\phi(x)\,,$$

$$(P\phi)(x) = \frac{1}{i}\,\frac{\partial}{\partial x}\,\phi(x)\,; \qquad (\hbar = 1)\,.$$

They satisfy the commutation relations $[Q, P] \subseteqq i$. Denote $\delta_{p,q} = \exp i(pQ + qP)$; $p, q \in \mathbb{R}$. Form the *-algebra Δ, generated by the unitary operators $\delta_{p,q}$ on $\mathscr{H}$ by taking the finite linear combinations of them, the *-operation is defined by $(\delta_{p,q})^* = \delta_{-p,-q}$ and the product rule is given by

$$\delta_{p,q}\delta_{p',q'} = \delta_{p+p',q+q'} \exp\left\{-\frac{i}{2}(pq' - qp')\right\}.$$

The operator norm closure $\bar{\Delta}$ of Δ is the CCR C^*-algebra, realized as a concrete C^*-algebra in $\mathscr{B}(\mathscr{H})$ (all bounded operators on $\mathscr{H}$). It is equivalent with the one considered in Refs. [1] and [2]. We take this algebra as the basic C^*-algebra for an algebraic formulation of quantum mechanics for finite systems.

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