Commun. math. Phys. 35, 235—252 (1974)

Matrix Moment Methods in Perturbation Theory, Boson Quantum Field Models, and Anharmonic Oscillators

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Received June 1, 1973

Abstract. A matrix moment problem is considered in connection with any $x^{2m}(m=2,3,4,\ldots)$ anharmonic oscillator as well as the $(:\phi^{2m}(x):g(x))_2$ $(m=2,3)$ field theory models, whose Rayleigh-Schrödinger perturbation expansions for the ground state eigenvalue are known to diverge. The approximants related to such a problem are proven to converge to the eigenvalue, when applied to an expansion of the Brillouin-Wigner type. These approximants, whose construction involves only matrix elements occurring in the Rayleigh-Schrödinger expansion, are the approximants of a J-type matrix continued fraction, i.e. the $[N-1,N]$ matrix Padé approximants. The explicit analytical expression of matrix continued fraction is found in the anharmonic oscillators case.

I. Introduction

The approximation problem for the ground state eigenvalue and eigenvector of a class of perturbation problems has been recently treated by McClary [1]. This class includes the $g(:\phi^{2m}(x):g(x))_2$ $(m=2,3)$ field theory models as well as the $x^{2m}(m=2,3)$ anharmonic oscillators. In both cases the Rayleigh-Schrödinger (R-S) perturbation expansion for the ground state eigenvalue $E(g)$ is known to be divergent as fast as $((m-1)n)!$, this last result being valid for arbitrary finite m [2]. For any model in the class mentioned above but the $(:\phi^6(x):g(x))_2$ it has been proven that the divergent R-S perturbation expansion is Borel summable to the exact solution [3, 4]; in addition, for the $m=2,3$ anharmonic oscillators the expansion is also Stieltjes summable [5]. This last result provides a strong approximation statement, since the Stieltjes method is equivalent to the convergence of the Method of Moments (i.e. Padé) approximants.

No rigorous approximation statement directly generated by a summation method was however known for any field theoretical model, nor for the x^{2m} anharmonic oscillators with $m>3$.

Now it has been shown in Ref. [1] that a monotonic sequence of approximants converging to $E(g)$ for g real and positive may be obtained,