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Classical Scattering with Long Range Forces*

Ira W. Herbst**

Department of Physics, The University of Michigan, Ann Arbor, Michigan, USA

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Abstract. We discuss the classical two-body scattering problem for potentials which decrease at infinity like $r^{-\alpha}$, $1 \geqq \alpha > 0$. We prove existence and uniqueness theorems for scattering orbits parametrized by their asymptotic data. Wave operators are constructed and their properties discussed. We also discuss and prove cluster properties of the S-operator.

I. Introduction

A few years ago, Buslaev and Matveev [1], following Dollard's [2] example in the case of the Coulomb potential, constructed generalized wave operators for the quantum two-body potential scattering problem with potentials which decrease slowly at infinity (i.e. faster than $r^{-\alpha}$ for some $\alpha > 0$). (See also Ref. [3] for $\alpha \geqq 3/4$.) This problem has also been attacked by Alsholm and Kato [4] who rederive the results in [1] with less restrictive assumptions.

In this paper we examine the same problem in classical mechanics. We feel that a more fundamental appreciation of many aspects of the quantum mechanical situation can be gained through an understanding of the corresponding classical problem.

Our formalism is essentially that of Simon [5] who considers the corresponding short range case. (See also Refs. [6, 7] where another point of view is taken.) Thus we consider the time development of the system directly in phase space and define a scattering state as one in which the orbit in phase space is unbounded both as $t \to +\infty$ and $t \to -\infty$. In Section II we prove existence and uniqueness of scattering solutions parametrized by certain asymptotic data. These are the asymptotic momentum and a three-vector describing how for apart the orbits are asymptotically.

In Section III we introduce additional assumptions concerning the behavior of the derivatives of the potential, $V(\boldsymbol{x})$, at infinity which enable us to construct a certain quasi-free time evolution $U_t^{(0)}$. The operator $U_t^{(0)}$ approximates the full dynamics, U_t, well enough asymptotically so that the transformations $\Omega_t = U_{-t} U_t^{(0)}$ converge on phase space as

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** Present address: Department of Physics, Princeton University, Princeton, N. J. 08540, USA.