Commun. math. Phys. 35, 181—191 (1974)

On the Connectedness Structure of the Coulomb S-Matrix*

Ira W. Herbst**

Department of Physics, the University of Michigan, Ann Arbor, Michigan, USA

Received July 30, 1973

Abstract. The forward direction singularity of the non-relativistic Coulomb S-matrix is examined and discussed. The relativistic Coulomb S-matrix to order α is shown to have a similar singularity.

I. Introduction

It is well known that for *short range* forces, the S-matrix describing the scattering of a (spinless) particle from a potential can be usefully split up into two pieces,

$$S(\boldsymbol{k}_1, \boldsymbol{k}_2) = \delta(\boldsymbol{k}_1 - \boldsymbol{k}_2) + t(\boldsymbol{k}_1, \boldsymbol{k}_2). \tag{1}$$

This decomposition is useful and natural because after removal of an energy conserving delta function, $t(\boldsymbol{k}_1, \boldsymbol{k}_2)$ is a smooth (indeed, often analytic) function of its arguments. The "no scattering" part of S, $\delta(\boldsymbol{k}_1 - \boldsymbol{k}_2)$, is called the "disconnected part" while $t(\boldsymbol{k}_1, \boldsymbol{k}_2)$ is the "connected part".

In Section II we calculate the explicit form of the Coulomb S-matrix, $S_c(\boldsymbol{k}_1, \boldsymbol{k}_2)$, and show that the decomposition (1) is far from natural. Indeed, in a sense to be defined more precisely, there is no delta-function component in S_c, and thus S_c is "totally connected". However, $S_c(\boldsymbol{k}_1, \boldsymbol{k}_2)$ does not have the structure of a connected part associated with a short range interaction. In fact as we will show, S_c is more singular than $\delta(\boldsymbol{k}_1 - \boldsymbol{k}_2)$!

In Section III we discuss the one photon exchange diagram for relativistic Coulomb scattering and show that the S-matrix to order α has a similar singularity in the forward direction.

II. Forward Direction Singularity in the Coulomb Amplitude

Although the explicit form of the Coulomb scattering amplitude has long been known, it was only in 1964 that Dollard [1] gave the correct time dependent description of the scattering process. We briefly state his results:

* Work supported in part by the National Science Foundation.

** Present address: Department of Physics, Princeton University, Princeton, N. J. 08540, USA.