Commun. math. Phys. 35, 173—179 (1974)

Dilation-Analyticity and Decay Properties of Interactions

D. Babbitt* and E. Balslev**

Department of Mathematics, University of California, Los Angeles, California, USA

Received July 16, 1973

Abstract. Let $H = H_0 + V$ be a Schrödinger operator on $L^2(\mathbb{R}^n)$. We show that the more dilation analytic V is, the slower it must decay at infinity.

1. Introduction

In the theory of the Schrödinger operator $H = H_0 + V$, various assumptions are made about the interaction V in order to be able to prove useful theorems about the spectral and scattering properties of the operator. Two assumptions which are often made are dilation analyticity assumptions (see [1] and [2]) and decay assumptions (see, for example, [4]). These usually have not occurred together (at least explicitly). It is the purpose of this paper to explore the interrelations between these two assumptions. In particular we will show that *the more dilation analytic V is, the slower it must decay at infinity.*

Our proof is based on a certain complex variable result (Lemma 3.2) which gives a sufficient condition for an analytic function defined in an angular sector to be 0. This is a consequence of the Phragmen – Lindelöf theorem and a theorem of Carlson.

2. The Main Theorem

We will denote by $\mathscr{H}$, the Hilbert space $L^2(R^n)$ of complex square integrable functions on $\mathbb{R}^n$. As usual, the inner product is defined by:

$$(\psi_1, \psi_2) = \int_{\mathbb{R}^n} \overline{\psi_1(x)}\, \psi_2(x)\, dx\,.$$

Also $\|\psi\|^2 \equiv (\psi, \psi)$. $\mathscr{H}_+$ will denote the completion of $C_0^\infty(\mathbb{R}^n)$ with respect to the norm $\|\psi\|_+ \equiv \|H_0\psi\| + \|\psi\|$ where H_0 is the usual self-adjoint

*, ** This Research was partially supported by * NSF grant no. GP-33696 X and ** NSF grant no. GP-36336.