Commun. math. Phys. 35, 141—150 (1974)

New Applications of the Calculus of Variations in the Large to Nonlinear Elasticity

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Received July 1, 1973

Abstract. Global methods of the calculus of variations and the infinite dimensional critical point theories of Morse and Ljusternik are applied to investigate the structure of the equilibrium states of thin flexible elastic plates under general body forces. The arguments used are equally applicable to broad classes of physical systems governed by nonlinear elliptic partial differential equations.

In this paper we take up the problem of determining the global structure of the equilibrium states of a thin elastic plate (of arbitrary shape) under the action of quite general body forces. We assume throughout that the deformations are governed by the well-known (nonlinear) von Karman equations, and we use a number of new and far-reaching ideas in the calculus of variations in the large to study the global structure of the solutions of these equations. Actually the von Karman equations have a particularly interesting Hamiltonian structure, so that one of the main purposes of this paper is the determination of the theoretical implications of this structure.

In papers [1] and [2], the author (together with Fife) applied some results from the calculus of variations in the large to study the buckling of a thin elastic plate under rather general edge conditions. In this paper we take up the general problem of studying the combined buckling-bending problem. Previous determinations of the structure of the resulting equilibrium states have been primarily restricted to rather local considerations (for example [3]). By studying the qualitative features of the nonlinear operator associated with the von Karman equations, we are able to obtain *global* results on the structure of the equilibrium states that perhaps explain some of the complicated phenomena observed experimentally.

Our paper is organized as follows: In Section I we formulate the mathematical problem to be considered and review the basic information concerning this problem already established in our earlier papers. In Section II we investigate those qualitative features of the von Karman

* Research partially supported by AFOSR grant 73-2437.