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Renormalization of Feynman Amplitudes and Parametric Integral Representation

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Abstract. A new substraction formula is presented to renormalize Feynman amplitudes written in Schwinger's integral representation.

The substractions are generated by an operator acting on the integrand, which only depends on the total number of internal lines but is completely independent of the structure of the graph.

This formulation is also valid for non-renormalizable theories and is shown to reduce to Zimmermann's R-operation for scalar theories. It satisfies in any case Bogoliubov's recursive formula and yields an explicit tool for actual computations of renormalized Feynman amplitudes with a minimal number of substractions.

I. Introduction

A procedure for extracting finite parts from divergent Feynman amplitudes with the requirements of Lorentz covariance, causality and unitarity was given several years ago by Bogoliubov and Parasiuk [1]. They introduced a substraction procedure, the so-called R operation, defined recursively over the graphs and the subgraphs and they proved this method to be equivalent to the addition of infinite counter terms in the Lagrangian. The proof of the B.P. theorem was then completed by Hepp [2] and recently Epstein and Glaser [3] reformulated the problem as a decomposition of distributions in retarded and advanced parts.

On the other hand, using the parametric integral representation [6], Appelquist [4] was able to give in closed form the value of a substracted Feynman amplitude. He proved in the case of renormalizable theories that his substraction scheme satisfies the B.P.H. recursive prescription. This proof was established by decomposing the families of subgraphs into forests. Later, this was proved by Zimmermann [5] to be, in the momentum space, the general solution to the recursion of B.P.H. even in the case of non-renormalizable theories.

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