Commun. math. Phys. 35, 101—112 (1974)

The Yang-Lee Distribution for a Class of Lattice Gases

John S. N. Elvey

Mathematics Department, University of Otago, Dunedin, New Zealand

Received August 30, 1972; in revised form June 1, 1973

Abstract. Classical lattice gases moving on a simple cubic lattice are considered. The lattice is assumed to grow only one-dimensionally. The gas particles have hard cores (of diameter greater than the lattice spacing) and are further subject to interactions of finite range and finite order. The interactions outside the hard cores may be represented as the components of a ν-dimensional vector, φ, which is initially allowed to be complex.

Using a transfer matrix technique, an asymptotic expression is obtained for the grand canonical pressure (at complex values of the inverse absolute temperature β and the fugacity z).

Let $\lambda_1 \dots \lambda_M$ denote the eigenvalues of the transfer matrix. Define φ to be a *D^*-interaction* if and only if the quotients, λ_j/λ_k, $1 \leqq j < k \leqq M$, regarded as functions of β, z (with φ fixed) are *nonconstant. In this paper it is assumed that there exists at least one allowable D^*-interaction.* With this assumption, the main result is that if F denotes the set of interaction vectors for which the distribution, Ω, of limit points of zeros of the grand partition function in the complex z-plane at fixed β (res. complex β-plane at fixed z) contains a domain, then F contains no product set $A_1 \times \cdots \times A_\nu$, $A_k \subset \mathbb{C}$, $1 \leqq k \leqq \nu$ unless one or more of the A_k consists of (at most) isolated points. This implies that the set of vectors for which Ω consists of arcs is dense in the set of all allowable vectors (in the usual topology for $\mathbb{C}^\nu$).

1. Introduction

Since Yang and Lee [1] formulated their theory of phase transitions and proved the celebrated "Circle Theorem", for ferromagnetic Ising models, much work has been done to obtain more general results. The greatest advance has been the extension of the circle theorem to quite general classes of "quasi-ferromagnetic" Ising and Heisenberg models, including those with suitable many-spin interactions [2].

The principles of the Yang-Lee theory apply to any statistical mechanical system. In all cases the physical behaviour of the model depends on the precise form of the distribution, say Ω, of limit points of zeros, in the complex plane, of the grand partition function [3].

For the quasi-ferromagnets, the circle theorem states (in effect) that Ω is a subset of the unit circle; so that Ω contains at most one positive