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Shock Waves in the Newman-Penrose Formalism

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Abstract. Gravitational shock waves of order $n=2$ are considered and their description in radiation coordinates is discussed. It is found that in such coordinates there exist necessarily (trivial) discontinuities of first derivatives of $g_{\mu\nu}$. The structure of the propagation relation for this form of the shock wave is derived and the discontinuities of the Newman-Penrose field variables are determined.

1. Discontinuities of the Newman-Penrose Field Variables

In the study of shock waves it has been found that besides the essential discontinuities of the field variables there are also trivial discontinuities which can be eliminated by appropriate transformations. In the usual treatment of shock waves it is assumed that these discontinuities have been eliminated.

In certain cases however this elimination may conflict with some other demand we put on the description of the field. The case we shall consider here in detail concerns a gravitational field described in radiation (Bondi) coordinates. More specifically we shall use the Newman-Penrose formalism [1] and we shall consider a shock wave which is essentially of order $n=2$ and propagates on the null-surface Σ determined by the equation $x^0=0$. We shall show that, because of the use of radiation coordinates, there will necessarily be present discontinuities of certain first derivatives of the metric.

Since the shock wave is (essentially) of order $n=2$, we shall have discontinuities of at least some of the scalars Ψ^A determining the Weyl tensor[1]. A priori we shall assume that certain first derivatives of the tetrad components U, X^κ, ω and ξ^κ ($\kappa=2, 3$) may also be discontinuous. Similarly we may have discontinuities of the rotation coefficients ϱ, σ, α, β, γ, λ, μ and ν, as these coefficients contain first derivatives of the tetrad components. A systematic use of the Newman-Penrose equations will allow us to determine the derivatives of the different field quantities which are really discontinuous.

[1] We are using the notation introduced in [1].