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A Variational Principle for Black Holes

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Abstract. It is shown that the initial data which gives rise to stationary black hole solutions extremizes the mass for a given angular momentum and area of the horizon. The only extremum of the mass for a given area of the horizon but arbitrary angular momentum is the Schwarzschild solution. In this case, and when the angular momentum is small, the extremum of the mass is a local minimum. This suggests that the initial data for the Schwarzschild solution has a smaller mass than any other initial data with the same area of the horizon. If this is the case, there is no possibility of proving the occurrence of naked singularities by methods suggested by Penrose and Gibbons. Together with Carter's theorem, the fact that the extremum is a local minimum indicates that the Kerr solutions are stable against axisymmetric perturbations.

Introduction

It has been known for some time that a star of more than about two solar masses can undergo gravitational collapse and produce a black hole. If the collapse is not exactly spherical, the black hole will be initially in an excited state and gravitational radiation will propagate out to infinity and across the event horizon into the black hole. The gravitational radiation reaching infinity will reduce the mass of the system as measured from infinity. [1, 2] while the radiation falling into the black hole will cause the area of the event horizon to increase [3, 4]. If the solution is axisymmetric, the angular momentum will be conserved. Even if the solution is not exactly axisymmetric, it is probable that not all the angular momentum can be radiated away. One would therefore expect the black hole would settle down finally to a stationary equilibrium state which minimized the mass and maximized the area of the event horizon for a given angular momentum. If the black hole is rotating, this stationary state must be axisymmetric [4–6] and be (t, ϕ) symmetric, i.e. invariant under $t \to -t$ and $\phi \to -\phi$ [7, 8]. It therefore seems reasonable on physical grounds that among all (t, ϕ) symmetric sets of initial data, those which give rise to stationary black hole solutions minimized the mass and maximize the area of the event horizon for a given angular momentum. In fact to determine the position and area of the event horizon, the boundary of the region from which it is not possible to