Commun. math. Phys. 33, 305—322 (1973)
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Entropy, Information and Quantum Measurements

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Received March 20, 1973

Abstract. The conditional entropy between two states of a quantum system is shown to be nonincreasing when a complete measurement is performed on the system. The information between two quantum systems is defined and is shown to be bounded above by the logarithmic correlation. This inequality is then applied to the measurement process. The entropy changes in the observed system and the measuring apparatus are compared with the information gain in the measurement.

1. Introduction

The object of this paper is to discuss the measurement process in quantum theory using the concepts of information and entropy. The starting point is a model of the measurement process introduced by von Neumann [1] which treats both the system to be observed (S) and the measuring instrument (M) as finite quantum systems. The aim is to obtain relations between the information obtained in the measurement and the entropy changes in S and M.

In § 2 the fundamental quantities of information, entropy and conditional entropy are introduced. The entropy used here is the statistical (or Gibbs') entropy, which cannot, in general, be identified with the thermodynamic entropy. The further development is based on an inequality for the conditional entropy which says that the conditional entropy is nonincreasing when a complete measurement is performed on the system (§ 3). The inequality is believed to hold also under a general measurement, and this conjecture can be shown to be equivalent to the conjecture that the entropy for a quasilocal quantum system is strongly subadditive [2]. The information between two subsystems of a quantum systems in a given state is defined, and from the inequality above it is proved to be bounded above by the logarithmic correlation (§ 4). This result is applied to the measurement process. If the system $S+M$ is closed, then the time evolution is a unitary transformation and the total entropy is conserved. Under the assumption that any eigenstate of the observable to be measured is conserved under the interaction of S and M, it is proved that the entropy of M increases by at least as much as the information gained during the measurement