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Markov Processes, Bernoulli Schemes, and Ising Model

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Abstract. We give conditions for the Bernoullicity of the ν-dimensional Markov processes.

1. Symbols and Definitions

Z^ν is the ν-dimensional lattice of the points with integral coordinates and $K = I^{Z^\nu} = \prod_{\xi \in Z^\nu} I$, $I = \{0, 1\}$, is the space of sequences of 0's and 1's labelled with the points $\xi \in Z^\nu$.

The space K is compact if endowed with the topology obtained as product of the discrete topologies on the factors I.

Similarly if $\Theta \subset Z^\nu$ we define the compact space $K_\Theta = I^\Theta = \prod_{\xi \in \Theta} I$.

We shall identify the elements $X \in K_\Theta$ as subsets of Θ: so that $X = (x_1, x_2 \dots x_p) \in K_\Theta$ means the sequence $X \in K_\Theta$ with values 1 in $x_1, x_2, \dots, x_p$ and 0 in $\Theta \setminus X$.

If $X \in K$ and $\xi \in Z^\nu$ we put $\tau_\xi X = X + \xi = (x_1 + \xi, x_2 + \xi, \dots)$ if $X = (x_1, x_2 \dots)$. The transformations $\tau_\xi : K \to K$ form a ν-dimensional group which we denote with the symbol τ; τ transforms Borel sets into Borel sets.

If μ is a Borel probability measure on K which is τ-invariant and $\Lambda \subset Z^\nu$ is a finite set (i.e. $|\Lambda| < \infty$), then we can define Borel measures

$$\mu_\Lambda(X, E), \qquad Q_\Lambda(E) \quad \text{on} \quad K_{Z^\nu \setminus \Lambda}$$

as

$$\mu_\Lambda(X, E) = \mu(\{Y \mid Y \in K;\ Y \cap \Lambda = X;\ Y \cap (Z^\nu \setminus \Lambda) \in E\}) \ E \subset K_{Z^\nu \setminus \Lambda}, \tag{1.1}$$

$$Q_\Lambda(E) = \sum_{X \subset \Lambda} \mu_\Lambda(X, E) = \mu(\{Y \mid Y \in K,\ Y \cap (Z^\nu \setminus \Lambda) \in E\}). \tag{1.2}$$

The Radon-Nikodym derivative, defined for $X \subset \Lambda$ and $Y \subset Z^\nu \setminus \Lambda$

$$\frac{\mu_\Lambda(X, dY)}{Q_\Lambda(dY)} = f_\Lambda(X \mid Y) \tag{1.3}$$