Commun. math. Phys. 33, 221—242 (1973)

Construction of Analytic, Unitary Scattering Amplitudes from a Given Differential Cross-Section: A Refined Analysis*

D. Atkinson

Institut voor Theoretische Natuurkunde, Groningen, Netherlands

P. W. Johnson and R. L. Warnock

Illinois Institute of Technology, Chicago, Illinois, USA

Received March 12, 1973

Abstract. We extend previous work concerning the construction of unitary scattering amplitudes that correspond to the scattering data at a given energy. The dispersive and absorptive parts are by construction analytic in $\cos\theta$ in the small and large Lehmann ellipses, respectively. The dispersive and absorptive parts obtained here, in contrast to those obtained before, are shown to have continuous derivatives on the boundary of their domains of analyticity. The continuum ambiguity in the determination of the scattering amplitude, which is associated with a lack of experimental information on the inelastic contribution to unitarity, is present here as well.

Section I: Introduction

The problem of determining the scattering amplitude f at a given energy, from the differential cross-section σ at that energy, has been considered by several authors recently [1]. In particular, Atkinson, Mahoux, and Yndurâin [2] have dealt with the problem of constructing a scattering amplitude $f(z)$ which is analytic in z, the cosine of the barycentric scattering angle, inside a certain unifocal ellipse. This amplitude must correspond to a specified differential cross-section $\sigma(z)$, which is analytic inside the ellipse; in addition, $f(z)$ must satisfy unitarity. The present work is a refinement of Ref. [2]; we discuss the same nonlinear equation as was treated there, but we do the analysis in a smaller Banach space and obtain stronger results.

For simplicity, we limit our discussion in the first two sections to the case of purely elastic unitarity. The results may readily be generalized to handle a fixed contribution to unitarity from inelastic channels. In Section III we treat the inelastic case explicitly, and discuss the continuum ambiguity.

* Work supported in part by the National Science Foundation and a NATO Research Grant.