Commun. math. Phys. 33, 187—205 (1973)
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Exact Dynamics of an Infinite-Atom Dicke Maser Model

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Received March 9, 1973

Abstract. We study the dynamics of the Dicke maser model in the limit as the number of atoms becomes infinite and the coupling constant between the atoms and the radiation field goes to zero. We find that the limiting Hamiltonian is integrable and obtain an explicit closed form for the unitary time evolution operators. As a corollary we show that in the limit the radiation emitted by the model is coherent in the sense by Glauber.

§ 1. Formulation of the Problem

We study the radiation emitted by a large system of atoms in a super-radiant state. We make the electric dipole approximation for each atom, supposing that the overall dimensions of the system are small compared to the wavelength of the emitted radiation. The radiation is supposed to consist of photons, but for notational simplicity we let the emitted particles be bosons of unspecified nature.

We set up the Hilbert space and Hamiltonian in the standard manner [6, 8, 15]. Each atom is described by a two-dimensional space $\mathbb{C}^2$, and the system of n atoms by $\otimes^n \mathbb{C}^2$. For the rth atom we introduce the spin operators $J^{(r)}$ acting on the rth component of $\otimes^n \mathbb{C}^2$ and satisfying the commutation relations

$$\begin{gathered}[J_1^{(r)}, J_2^{(r)}] = i J_3^{(r)}; \quad [J_2^{(r)}, J_3^{(r)}] = i J_1^{(r)}; \quad [J_3^{(r)}, J_1^{(r)}] = i J_2^{(r)}; \\ J_+^{(r)} = J_1^{(r)} + i J_2^{(r)}; \quad J_-^{(r)} = J_1^{(r)} - i J_2^{(r)}; \quad [J_i^{(r)}, J_j^{(s)}] = 0 \quad \text{if} \quad r \neq s. \end{gathered} \tag{1.1}$$

The single particle space for the emitted radiation is denoted by $\mathscr{H}$ and the quantised radiation field is the boson Fock space $\mathscr{F}$ over $\mathscr{H}$:

$$\mathscr{F} = \mathbb{C} \oplus \mathscr{H} \oplus \{\otimes_{\text{sym}}^2 \mathscr{H}\} \oplus \{\otimes_{\text{sym}}^3 \mathscr{H}\} \oplus \cdots. \tag{1.2}$$

For any $f, g \in \mathscr{H}$ we have smeared creation and annihilation operators $a^*(f)$ and $a(g)$ on $\mathscr{F}$ with commutation relations

$$\begin{gathered} a(g)\, a^*(f) - a^*(f)\, a(g) = \langle f, g \rangle 1, \\ [a(g), a(f)] = 0. \end{gathered} \tag{1.3}$$