Commun. math. Phys. 33, 145—164 (1973)

The $(\phi^4)_2$ Field Theory as a Classical Ising Model

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Received May 16, 1973

Abstract. We approximate a (spatially cutoff) $(\phi^4)_2$ Euclidean field theory by an ensemble of spin 1/2 Ising spins with ferromagnetic pair interactions. This approximation is used to prove a Lee-Yang theorem and GHS type correlation inequalities for the $(\phi^4)_2$ theory. Application of these results are presented.

1. Introduction

Rigorous tools in the theory of the Ising model fall roughly into two classes. Certain results have only been proven directly for what we will call "classical Ising models" by which we mean ferromagnets with spin 1/2 (i.e. s_i can have the values ± 1) spins and pair interactions. Others hold for "general Ising models" by which we mean that arbitrary (ferromagnetic) many body interactions are allowed and that individual spins can take an arbitrary set of values (including continuous values) with fairly arbitrary uncoupled single spin probability distributions. The first class includes the zero theorem of Lee and Yang [15, 1] and the correlation inequalities of the Griffiths-Hurst-Sherman (GHS) type [8]. The second class includes the correlation inequalities of Griffiths, Kelly, and Sherman (GKS) [6, 13] and of Fortuin, Kasteleyn, and Ginibre (FKG) [3]. (GKS inequalities were originally proven for classical models [6] but were eventually proven with many body interactions [13], higher spin [7] and arbitrary even spin distributions [4].)

Recently, Guerra, Rosen and Simon [12] have shown how the $P(\phi)_2$ Euclidean field theory [16, 12] can be approximated by general Ising models. As a consequence of this approximation (which they called the "lattice approximation") they were able to prove GKS and FKG inequalities. Our goal in this paper is to investigate the possibility of approximating such field theories by classical Ising models.

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