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On the Role of the Killing Tensor in the Einstein-Maxwell Theory

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Abstract. It is shown that when a four dimensional source-free Einstein-Maxwell space-time admits a group of motions leaving the electromagnetic field unchanged a linear relation exists between two Maxwell fields and the covariant derivative of a Killing vector. For the case in which the two electromagnetic fields are related by a duality rotation it is seen that a purely geometric form of Einstein's equations may be derived. The behaviour of these under a class of quasi conformal transformations of the metric is shown to lead to Harrison's theorem.

1. Introduction

While the geometric nature of the Einstein-Maxwell electromagnetic field is now well established (Rainich, 1925; Misner and Wheeler, 1957) there are nevertheless some aspects which are not very well understood. A particular case in point is the fact that solutions of the equations may be generated both from known solutions and from solutions of the empty-space equations $G_{\mu\nu}=0$ when a symmetry is present (Harrison, 1965 and 1968).

The object of the present paper is to establish a result which appears to be of relevance in explaining solution generation and the role which the Killing vector has in the transformation properties of the field equations.

2. Einstein's Equations

We shall consider a four dimensional space-time which satisfies the vacuum Einstein-Maxwell equations. These may be written

$$R_{\mu\nu}=4\pi(F_{\mu\sigma}F^{\sigma}_{\nu}+{}^*F_{\mu\sigma}{}^*F^{\sigma}_{\nu}) \tag{2.1}$$

and

$$\left.\begin{aligned} F^{\mu\nu}{}_{|\nu}&=0\\ {}^*F^{\mu\nu}{}_{|\nu}&=0 \end{aligned}\right\} \tag{2.2}$$

where the vertical bar denotes covariant differentiation.