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Potentials for almost Markovian Random Fields

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Abstract. A positive almost Markovian random field is a probability measure on a lattice gas whose finite set conditional probabilities are continuous and positive. We show that each such random field has a potential and in the translation invariant case an absolutely convergent potential. We give a criterion for determining which random fields correspond to pair potentials, or in general n-body potentials. We show that two translation invariant positive almost Markovian random fields have the same finite set conditional probabilities if and only if one minimizes the specific free energy of the other.

1. Introduction

There are two natural approaches to the statistical properties of a lattice gas. The interaction can be described either in terms of conditional probabilities or in terms of potentials. For nearest neighbor pair interactions Averintsev [1] and Spitzer [11] showed these two approaches to be equivalent. Then Averintsev [2], Sherman [10], and Sullivan [12] extended this analysis to finite range interactions.

The type of interaction potential considered by Gallavotti *et al.* [5, 7] suggests an extension of the finite range results. Such potentials give rise to measures whose conditional probabilities are limits of conditional probabilities on finite subsets of the lattice. In the compact totally disconnected configuration space this amounts to the requirement of continuity for certain conditional probabilities. Interestingly, the resulting class of probability measures is slightly larger than the class of probability measures strictly associated with the interaction potentials mentioned above.

Nevertheless, we can find potentials with weaker convergence properties and the method of finding these potentials provides a criterion for determining whether a given probability measure corresponds to a potential involving n or fewer particles at a time.

With the additional assumption of translation invariance we can get better convergence for the potential and can prove the existence of specific free energy. We give a proof based on conditional probability of a specific free energy variational theorem.