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States and Dynamics of Infinitely Extended Physical Systems

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Abstract. A scheme is presented for the description of the states and dynamics of infinitely extended systems. In this scheme, the physical states of a system are taken to comprise the maximal folium of its locally normal states which can support a one-parameter group of affine transformations, that corresponds to a certain infinite volume limit of the time-translational group for the states of a finite system of particles of the same species. The resultant one-parameter group of transformations of the physical states of the infinitely extended system is then taken to correspond to its time-translations. An explicit construction is given which serves to identify the physical states and dynamics of the system in terms of its interactions. The present scheme generalises that of Dubin and the author beyond the islands of Gibbs states.

1. Introduction

It is well-known that the generalisation of quantum or classical dynamical laws to infinitely extended physical systems presents certain problems (cf. [1, 2, 3]). A first approach to a dynamical theory for such systems was made within the C^*-algebraic framework by Haag, Hugenholtz, and Winnink (HHW) [4]. The scheme of HHW was based on a postulate concerning the existence of a certain "infinite volume limit," which led to a description of time-translations of an infinite system in terms of a one-parameter group of automorphisms of the C^*-algebra of its observables. However, although the basic postulate of HHW is satisfied by lattice systems with suitably tempered interactions [5], it is not generally valid for non-relativistic continuous systems [1] or for lattice systems with sufficiently long range interactions [2].

A subsequent approach to the dynamical problem by Dubin and the present author (DS) [1], based on weaker assumptions than those of HHW, led to a description of time-translations in the "island" of a Gibbs state in terms of a one-parameter group of automorphisms of the weak closure of the associated GNS representation of the algebra of observables. However, although the DS scheme is applicable to a wider class of systems than that of HHW, it has the disadvantage of being strictly limited to the islands of Gibbs states. This limitation is serious