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Asymptotic Orbits in a Free Fermi Gas

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Abstract. We consider the time evolution of local observables and physical states in an infinite system of non-interacting Fermi particles. The orbit of an observable in the C^*-algebra of the canonical anticommutation relations is proved to be asymptotic to a set of observables consisting of sums of products of elements of grade two and lower with support in a family of separated cells in $\mathbb{R}^3$ (a *lacunary paving* of $\mathbb{R}^3$) under time evolution. A space-factorization ("clustering") property for primary, even, locally Fock states is established. A class of such states whose space-correlations decay as $(\log d)^{-(1+a)}$ with a positive and d the (space-) separation is, then, proved to be time-asymptotic to their associated quasi-free states.

§ 1. Introduction

Statistical mechanics results from a "coarse grained" description of a system, i.e., from the process of limiting attention to a restricted set of observables. It has been the hope underlying many recent studies in Quantum Statistical Mechanics that, for most purposes, it is adequate to take for this set the local observables in an infinitely extended medium. If this is so then we should be able to show that time evolution of states over the algebra of local observables in an infinite system is such that a considerable simplification results as the time t tends to infinity (insensitivity to initial conditions): the orbits of large classes of states should coalesce (become asymptotic to each other) as $t \to \infty$. While it appears forbiddingly difficult to obtain any rigorous mathematical results about asymptotic orbits of states for systems of infinitely many *interacting* particles, it is possible to study this problem for the *free* Fermi gas (an infinite system of non-interacting particles). From the physical point of view, the free system is, of course, a poor example to demonstrate the asymptotic simplification because interparticle collisions