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Exponential Decay of Bound State Wave Functions*

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Abstract. The spatial decay properties of the wave functions of multiparticle systems are investigated. The particles interact through pair potentials in the class $R+L^\infty_\varepsilon$. The bound states lie below the bottom of the continuous spectrum of the system. Exponential decay, in an L^2 sense, is proven for these wave functions. The result is the best possible one which will cover every potential in this class.

Introduction

In every exactly soluble quantum mechanical two body problem the bound state wave functions fall off exponentially. The rate of exponential decay depends only on the energy of the bound state and the masses of the particles involved. For several particles intuitive arguments on bound states near the bottom of the continuum suggest that for these systems too the wave functions decay exponentially and that the rate of decay depends only on the particle masses and the depth of the bound state below the bottom of the continuum [15]. These L^2 wave functions are initially just in the domain of the system Hamiltonian. When the potentials can be written as the sum of a bounded function and an L^2 function with compact support Kato [8] has shown that the wave functions are actually bounded Holder continuous functions. Hunziker [7] and Simon [14] have observed that one way to state a decay result is as an L^2 domain condition.

For the class of potentials considered by Kato, Ahlrichs [1] has shown that L^2 conditions imply pointwise decay. This rate of decay is weaker than the L^2 decay but Simon [25] has recently shown that for a slightly different class of potentials the pointwise decay is precisely the same as the L^2 decay. We will prove the following theorem.

Theorem. *Scalar particles with masses m_i $(1 \leqq i \leqq N)$ interacting through local potentials $V_{ij} \in R+L^\infty_\epsilon$ have a bound state at energy $-E$, $E>E_0$*

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