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Extensions of the Curzon Metric

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Abstract. By examining the behaviour of geodesics approaching the singularity of the Curzon solution, it is shown that the metric is capable of being extended in such a way that almost all such geodesics are complete. There are an infinite number of possible extensions. None are analytic, but all are C^∞.

1. Introduction

The general static axially symmetric vacuum solution of Einstein's field equations is given by the Weyl metric

$$ds^2 = -e^{2(\nu-\lambda)}(dr^2 + dz^2) - r^2 e^{-2\lambda} d\phi^2 + e^{2\lambda} dt^2 \tag{1}$$

where

$$\lambda_{rr} + \lambda_{zz} + r^{-1}\lambda_r = 0,$$

and

$$\nu_r = r(\lambda_r^2 - \lambda_z^2), \qquad \nu_z = 2r\lambda_r\lambda_z.$$

We may pick for λ any finite multipole expansion in inverse powers of $R = \sqrt{r^2 + z^2}$, thus generating a whole range of "particle-like" solutions which have some interesting properties [1]. Israel's theorem [2] indicates that none of these solutions has a regular event horizon at $R = 0$, for the Schwarzschild solution appears as a rod of length $2m$ in Weyl coordinates and hence does not belong to the family. One should however be wary of the applicability of Israel's theorem even in this simple case for the equipotential surfaces are by no means always regular and may exhibit cusps or may even break up into two or more disjoint pieces. Furthermore it still remains an unsolved problem whether a realistic collapse of a spherically asymmetric system may or may not result in such a naked singularity [3].

It is also not at all clear that the $R = 0$ singularity is in general "point-like" [4]. Indeed if one considers the monopole solution (the so called Curzon metric),

$$\lambda = -\frac{m}{R}, \qquad \nu = -\frac{m^2 r^2}{2R^4}, \tag{2}$$

then the behaviour of invariants of the curvature tensor in the limit $R \to 0$ is strongly dependent on the direction of approach. Gautreau