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Instability of Closed Spaces in General Relativity

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Abstract. There are strong restrictions on the solutions of the initial value constraints of General Relativity when the spatial hypersurface is closed. In particular, closed flat space is unstable: not all solutions of the linearized constraints correspond to nearby solutions of the constraints themselves. For example, no nearby solutions whatever exist which are time symmetric. Other restrictions, which limit perturbations of non-flat closed initial solutions, are also exhibited.

I. Introduction

Although General Relativity may be characterized as a massless spin two Lorentz covariant field theory, its non-linearities can lead to qualitative complications not encountered in other field theoretical models. Most evident is the existence of solutions describing closed universes, in which the usual asymptotic falloff at infinity is absent. We shall investigate here some implications of the initial value equations peculiar to closed universes. The basic point is that flux integrals vanish identically when there is no boundary. In electrodynamics, for example, the total charge in a closed universe must be zero (because it can be expressed as a surface integral). The analogous gravitational quantity in asymptotically flat spaces is the total energy, which also can be expressed as a surface integral. The energy is due not only to sources but also to the gravitational field itself, and it has positive definiteness properties in its field dependence. Thus, if the mass is required to vanish, no excitations whatever are allowed. Below we investigate the restrictions which the

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