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Analyticity of Correlation Functions in One-Dimensional Classical Systems with Slowly Decreasing Potentials

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1. Introduction

Four years ago, the following result was proved simultaneously by the author of the present paper [1–3] (for the case of general classical lattice systems), and by Ruelle [4] (for classical lattice gases). If the total energy of interaction between particles on the right-half line and particles on the left half-line is bounded, then the infinite-volume limits of the correlation functions exist, and the infinite-volume correlation functions have some rather strong cluster properties. (In the language of probability theory [5], this property is known as "uniform strong mixing" of the corresponding random processes.) The examples constructed later by Dyson [6] and by Fisher and Felderhof [7] show that the condition on the potential cannot be essentially weakened. These results have been extended to continuous systems with hard cores by the author [3] and by Gallavotti, Miracle-Sole, and Ruelle [8, 9].

The above results are interpreted physically as implying the absence of phase transitions, and so it is natural to conjecture that, in these cases, thermodynamic and correlation functions vary analytically with the parameters in the interaction. For classical lattice systems with finite-range potentials such analyticity follows easily from standard theorems of linear algebra (cf. [10]), and for continuous systems with hard cores and finite-range potentials, this result was proved by van Hove [11] (see also [10]). Recently Araki proved this result for lattice systems with exponentially decreasing potentials [12]. Under similar conditions on the potential, this result can be obtained as a consequence of some results of probability theory (Statulevičius [13], Zhurbenko [14]), Gallavotti and Lin [15] proved that for potentials vanishing at infinity more rapidly than $\exp\{-r^\alpha\}, \alpha > 0$, the thermodynamic and correlation functions are infinitely differentiable. For potentials vanishing only like an inverse