Commun. math. Phys. 32, 231—243 (1973)

The Smallest C^*-Algebra for Canonical Commutations Relations

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Received February 7, 1972; in revised form March 23, 1973

Abstract. We consider the C^*-algebras which contain the Weyl operators when the symplectic form which defines the C.C.R. is possibly degenerate. We prove that the C.C.R. are all obtained as a quotient of a universal C^*-algebra by some of its ideals, and we characterize all these ideals.

I. Introduction

In a recent paper [1] Slawny derived the following very interesting result.

There exists a C^*-algebra $\mathfrak{A}$ which is such that to every representation (not necessarily continuous) of the C.C.R. there corresponds a representation of $\mathfrak{A}$; moreover, $\mathfrak{A}$ is simple and minimal.

Non degeneracy of the symplectic form which defines the C.C.R. (see below) seems essential to his derivation; in this paper, we shall not make this assumption and through a quite different approach we shall be able to give a description of the C^*-algebras which contain the Weyl operators.

We define a universal C^*-algebra which coincides with the one defined by Manuceau [2] and Slawny [1] in the case where the symplectic form of the C.C.R. is non degenerate; the definition is specially simple to handle and in particular we prove that any positive linear form on finite combinations of Weyl operators extends to a state of this algebra.

Moreover any C^*-algebra containing the Weyl operators is the quotient of the universal one by an ideal which is in some sense characterized by its intersection with the center of the algebra.

A section is devoted to the study of central states, and our results are close of Slawny in the case of non degeneracy.

Finally we make the following remark: Degeneracy of the symplectic form which defines the C.C.R. has been encountered already in the study of quasi-free bose gas below the critical temperature [3] and is possibly interesting to consider in the study of field theory with massless particles.