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# Equilibrium States of a Dimer Model with Angular Forces

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**Abstract.** Several complementary techniques are applied to the study of the orientational transition in a restricted lattice model of rigid linear dimers with finite interactions between contiguous molecules, on the square lattice. The restriction has the effect of forcing the ordered phase to resemble either a smectic or a nematic liquid crystal. It is shown that the symmetry of the equilibrium state is broken for some interactions and that the equilibrium state is unique for others. Thermodynamic analyticity is established for high temperatures.

## 1. Introduction

Considerable effort has been expended in the study of dense fluids and their phase transitions. There has also been considerable interest in various models of liquid crystalline systems. One of these models is the continuum rigid rod model which has the virtue of relative mathematical simplicity. The combination of these two classes of models yields a model of rigid straight polymeric molecules that are constrained to fit on some lattice; dimers on a square lattice represent the simplest example. This system is known to have no phase transition (Heilmann and Lieb [1], Gruber and Kunz [2], Runnels and Hubbard [3]) when all allowed configurations of a specified number of molecules have the same energy. If this restraint is removed and intermolecular interactions permitted, the possibility of an orientational phase transition must be reexamined. It will be shown that for a restricted version of the model, and some interactions, a phase transition does occur. When we refer to the existence of a phase transition, we are actually referring to a breakdown of symmetry in the equilibrium states as discussed by Dobrushin [4] and Lanford and Ruelle [5]. We do not attempt to infer from this symmetry breakdown anything about nonanalyticity of the free energy.

## 2. The Restricted Dimer Model

The underlying lattice for this problem is the two-dimensional square lattice. On that lattice we shall place straight rigid dimers whose width is the edge of the unit cell of the square lattice. The dimer may be of