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Geometry in a Manifold with Projective Structure*

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Abstract. Parallel transport of line elements, surface elements etc. along geodesics and more general curves in a projectively connected manifold is investigated analytically and in terms of geometrical constructions. Projective curvature is characterized geometrically by a projective analogue of the geodesic deviation equation and by a geometrical construction. The results are interpreted physically as statements about free fall world lines in space-time.

I. Introduction; Geometry of Free Fall and Light Propagation

The theory of *special relativity*, developed by Lorentz, Poincaré, and Einstein in the years 1902–1905 and perfected by Minkowski in 1908 has as its basic ingredient the properties of *light propagation*. It is these properties which suggest the ideas of an absolute limiting speed, the behavior of time (no absolute simultaneity, clock effect), and finally the full geometrical edifice of Minkowski space-time.

Einstein's theory of *general relativity* of 1915 retains all the concepts of special relativity for infinitesimal space-time regions, and supplements them with a new basic ingredient, the properties of *free fall*. It is these properties which suggest the ideas of the principle of equivalence, the (pseudo-) Riemannian structure of space-time, and finally Einstein's gravitational field equations.

In a recent paper [1], Pirani and the authors have shown that the space-time structure of general relativity theory can be fully explored by observing the free fall of particles and the propagation of light. We give a short summary of these results (which are illustrated in Fig. 1):

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