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Normal-Dominated Singularities in Static Space-Times

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Abstract. We define "normal-dominated" singularities of static solutions of the Einstein equations and show that a uniquely and invariantly defined structure can be assigned to these singularities. We find for the general solution that the dominant term of the Riemann tensor near the singularity is of Petrov Type N. Except for one special class of solutions, it seems that in general the shear of the null geodesics blows up at the same rate as their convergence near the singularity, in contradistinction to the "elementary singularity" of Newman and Posadas. We compute the structure for a variety of known static solutions as well as the stationary Kerr-Newman metrics.

I. Introduction

The singularity theorems of Hawking, Penrose and others [1–4] indicate that, assuming reasonable properties for the matter source and global structure, singularity in general relativistic space-times is both a general and stable phenomenon. It is then natural to inquire about the nature and structure of these singularities. For the sake of physical interest, we will consider only the type for which curvature or matter variables blow up. In cosmological models[1], the works of Lifshitz, Khalatnikov and Belinskii [5], and of Eardley, Liang and Sachs [6] seem to indicate that the singularities are of two types, namely, the velocity-dominated and the mix-master. Moreover, in the velocity-dominated case, a unique and invariant structure can be assigned to the singularity. What about other space-times?

In this paper we are going to show that a similar construction can be worked out for the class of static space-times which we call "normal-dominated." In a static[2] space-time, the singularity, if it exists, must be in some sense "time-like," since it must be tangent to the time-like

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[1] By a cosmological model we mean a space-time which admits no time-like Killing vector and contains some form of macroscopic matter source.

[2] A space-time is static if it admits an irrotational Killing vector every-where time-like except maybe at the singularities. Thus Schwarzschild solution is not static near the singularity $r = 0$.