

Highly Mobile Einstein Spaces in the Large

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Abstract. We consider an Einstein space V of the Petrov type II or III admitting a group of motions G of high order. First we calculate the composition law and topological structure of G . Then V (or its submanifolds of transitivity) is represented as the homogeneous space G/H of G , H being a subgroup of G , and the action G on V and the topology of V are determined. The topologies of the spaces V are as follows: \mathbb{R}^4 (space T_2^*), \mathbb{R}^4 of $\mathbb{R}^3 \mathbb{T}^1$ (space T_2), \mathbb{R}^4 (space T_3^*), \mathbb{R}^3 (submanifolds of transitivity in space T_3).

In two cases (spaces T_2 and T_3) we have obtained metrics free of singularities.

§ 1. Introduction

The aim of this work is to investigate the global structure of some Einstein spaces T_i (satisfying the field equations $R_{\alpha\beta} = 0$) and T_i^* (satisfying the field equations $R_{\alpha\beta} = \kappa g_{\alpha\beta}$, $\kappa \neq 0$) possessing high mobility. Here $i = 1, 2, 3$ is the Petrov type.

The *local* part of this problem has been solved by Petrov ([1], Chapter 5) who has determined Lie algebras of Killing vectors and metrics of these spaces. To obtain a *global* information about the spaces under consideration we shall use the new topological methods based on the idea of homogeneous spaces of Lie groups (see, e.g., [2]).

At present a global investigation of gravitational fields is a matter of current interest. It is particularly interesting to examine the topology of the highly mobile Einstein spaces $T_2(T_2^*)$ and $T_3(T_3^*)$ since:

- (i) these spaces are often interpreted as gravitational waves;
- (ii) these spaces cannot be asymptotically flat and so they are assumed to have peculiar topology ([1], § 30);
- (iii) the boundary conditions for Einstein spaces of the second and the third types are formulated in terms of the highly mobile Einstein spaces of the same types ([1], § 65) so that topology of arbitrary Einstein spaces of the Petrov type II or III is determined by the topology of the corresponding Einstein spaces having maximal mobility.

Taking this into consideration we shall investigate below the global structure of the Einstein spaces T_2 , T_2^* , T_3 and T_3^* , possessing maximal mobility.