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The Space of Lorentz Metrics*

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Abstract. The set of all C^2 Lorentz metrics on a non-compact four-manifold is given the Whitney fine C^2 topology. It is shown that this provides the correct framework within which to discuss the global properties of spacetime manifolds in general, and the singularity theorems in particular. The main result is a theorem showing that the Robertson-Walker big bang (global infinite density singularity in the finite past) is stable under sufficiently small, but otherwise arbitrary, finite C^2 perturbations of the metric tensor.

I. Introduction

This paper deals with the topological structure of the space of all exact solutions to the Einstein field equations on an arbitrary four-manifold. A rigorous mathematical framework emerges within which it is possible to pose and answer the following questions:

(1) Which of the well-known global properties of spacetimes are stable under sufficiently small perturbations of the metric tensor?

(2) Do the singularity theorems of Hawking, Penrose and others translate into statements concerning the topology of this space? For instance, are G-singularities *stable* in the set of "physically realistic" metrics (do they form an open set)? Are there situations in which G-singularities are *generic* (for example, when the spacetime contains a closed space section, or when it contains an object undergoing catastrophic gravitational collapse)?

(3) Do there exist situations in which it is possible to say something precise about the exact nature of the singularity (other than that an incomplete causal geodesic exists)? For example, *is the "big bang" stable*? If a Robertson-Walker model is slightly perturbed, does the new spacetime still possess an infinite density singularity over all space in the finite past?

Section II presents the necessary mathematical machinery; it reviews jet bundles, defines and to some extent motivates the Whitney mapping

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