Commun. math. Phys. 31, 295—316 (1973)

Relativistic Quantum Theory for Charged Spinless Particles in External Vector Fields

Lars-Erik Lundberg
Nordita, Copenhagen, Denmark

Received February 5, 1973

Abstract. Guided by a diagonalized form of the classical field-energy we construct a time-dependent canonical pair of Schrödinger fields $\Phi_t(x)$ and $\Pi_t(x)$ which diagonalizes the field-Hamiltonian H_t. These Schrödinger fields in general belong to inequivalent representations of the canonical commutation relations for different t's.

The Heisenberg field is constructed by solving the Heisenberg equation of motion and its time-evolution turns out to be governed by a unitary operator, i.e. the Heisenberg fields at different times are unitarily equivalent.

Scattering theory (including eventual incoming and/or outgoing bound-states) is finally constructed.

I. Introduction

We shall in this paper develop a Hamiltonian formulation of relativistic quantum theory for charged spinless bosons in a local external vector potential $A_\mu(x, t)$. The formulation will be free from divergences.

External field problems in relativistic quantum theory has been studied frequently in the past and one naturally asks oneself if anything new can be added. In order to get a motivation for this work, let's briefly summarize what previously has been achieved.

Time-independent External Vector Potential: The solution of the external field problem in the time-independent case was essentially given by Heisenberg and Pauli [1] in their classical paper on quantum field theory. They proposed that one should quantize a classical field by expanding it in terms of the stationary solutions (eigenfunctions) and quantizing the normal coordinates (generalized Fourier-components). They actually only considered the Dirac case (spin 1/2), but the Klein-Gordon case (spin 0) can be treated in complete analogy, see Schnyder and Weinberg [2].

The Hilbert-space $\mathscr{H}$, on which the fields are realized, is the Fock-space associated with the stationary modes, and the field-Hamiltonian H and the charge-operator Q define diagonal self-adjoint operators in $\mathscr{H}$ (when properly normal ordered) provided $A_\mu(x)$ is sufficiently regular.